

# Math Diversion Problem 790

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September 8, 2025

Math is just hard.  
— Alex Wei

Source: <https://www.youtube.com/watch?v=qNXppQan-WQ>  
Title: ODE with exponential solution  
- Oxford Mathematics Admissions Test 2018  
Presenter: Math Out Loud

## 1 Problem

The function  $y = e^{kx}$  satisfies the equation

$$\left(\frac{d^2y}{dx^2} + \frac{dy}{dx}\right) \left(\frac{dy}{dx} - y\right) = y \frac{dy}{dx}, \quad (1)$$

for how many values for  $k$ ?

## 2 Solution

We can quickly convert the given equation to an algebraic equation because  $y = e^{kx}$ , so that

$$\frac{dy}{dx} = ky \quad \text{and} \quad \frac{d^2y}{dx^2} = k^2y, \quad (2)$$

so substituting these values into (1), we have that

$$(k^2y + ky)(ky - y) = ky^2. \quad (3)$$

Can we simplify this by dividing through by  $y^2$ ? We can if  $y \neq 0$ , but we know that this is true because  $y = e^{kx}$  and this is never zero. Hence, we have that

$$(k^2 + k)(k - 1) = k. \quad (4)$$

So  $k = 0$  is one solution. When  $k \neq 0$  there are other possible solutions. Divide through by  $k$ , to get

$$(k + 1)(k - 1) = 1. \quad (5)$$

This last equation has roots  $k = \pm\sqrt{2}$ , making a total of three solutions for  $k$ .