Math Diversion Problem 798

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It is often the case that getting
organized is half the battle.
— The Author

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Source: https://www.youtube.com/watch?v=aCGPeRC9RRk
Title:  How To Solve A Beautiful Exponential Equation
 Presenter: SyberMath
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1 Problem

Given the relation

$$4^x + 4^{1/x} = 18\,, \tag{1}$$

solve for the real values of x.

2 Solution

First, rewrite the given to the form

$$4^x + 4^{x^{-1}} = 18\,, \tag{2}$$

then write

$$\frac{1}{2}4^x + \frac{1}{2}4^{1/x} = 9\,. \tag{3}$$

Hey, we might be able to get one or two integers solutions, and two real solutions is the most we can hope for. So, I'm guessing that either the first term is 8 and the second is 1, or the first term is 1 and the second term is 8. And guess what? We can get both of those cases by choosing

$$x = 2 \quad \text{and} \quad x = 1/2\,. \tag{4}$$