# Math Diversion Problem 801

P. Reany

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Cohomology isn't just one tool;
it's the central tool.
— Colin McLarty

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Source: The Ether of Great Mathematical Ideas
Title:  The Quotient Rule
 Presenter: Patrick
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## 1 Problem

I'd like to prove the quotient rule of calculus using logarithmic differentiation. It isn't new, but maybe you haven't see it yet.

Given the relation

$$\phi(x) = \frac{f(x)}{g(x)}\,, \tag{1}$$

where $f(x) \neq 0, g(x) \neq 0$ are differentiable functions, show that

$$\phi'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}\,. \tag{2}$$

## 2 Solution

There are many ways to prove this, but logarithmic differentiation is instructive, if for no other reason than because the technique comes up in physics, chemistry, and economics.

So, we begin by taking the logarithm across (1), to get[1]

$$\log \phi(x) = \log f(x) - \log g(x)\,. \tag{3}$$

Next, we differentiate through:

$$\frac{\phi'(x)}{\phi(x)} = \frac{f'(x)}{f(x)} - \frac{g'(x)}{g(x)}\,. \tag{4}$$

[1]The base we use to take this logarithm isn't important because, in the end, there won't be any logarithms left.

Now, we multiply through by $\phi(x)$ from (1), to get

$$\phi'(x) = \frac{f'(x)}{g(x)} - \frac{f(x)g'(x)}{g(x)^2}\,. \tag{5}$$

And finally, we can write

$$\phi'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}\,. \tag{6}$$