Math Diversion Problem 802

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> Thermodynamics is Nature's way of balancing
> entropy with enthalpy.
> — Rafael Jaramillo

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Source: The Ether of Great Mathematical Ideas
Title:  The Hidden Quadratic Equation
 Presenter: Patrick
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1 Problem

Given the relation

$$2^x + 12(2)^{-x} = 7\,, \tag{1}$$

solve for the real values of x.

2 Solution

The trick is to see a quadratic in the palm of your hand. So, let's rewrite the Given to

$$2^x + 12(2^x)^{-1} = 7\,, \tag{2}$$

and then multiply through by 2^x, to get

$$(2^x)^2 + 12 = 7(2^x)\,. \tag{3}$$

But this can be put into standard form, as

$$(2^x)^2 - 7(2^x) + 12 = 0\,, \tag{4}$$

which is a quadratic in variable 2^x, which can be factored to

$$(2^x - 3)(2^x - 4) = 0\,, \tag{5}$$

with roots

$$2^x = 3 \quad \text{or} \quad 2^x = 4 = 2^2\,. \tag{6}$$

Therefore, x has solutions

$$x = \frac{\log 3}{\log 2} \quad \text{or} \quad x = 2\,. \tag{7}$$