

Math Diversion Problem 805

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First things first...But not necessarily in that order.

— Doctor Who

Source: The Ether of Great Mathematical Ideas

Title: Attack of the square roots

Presenter: Patrick

1 Problem

Given the relation

$$x = \sqrt{x}^{\sqrt{x}} \quad \text{where } x > 0, \quad (1)$$

find the solutions for x .

2 Solution

Let's simplify by letting

$$x = y^2. \quad (2)$$

Then (1) becomes

$$y^2 = y^y. \quad (3)$$

Next, we take the logarithm across this, to get

$$2 \log y = y \log y. \quad (4)$$

One solution for this is $y = 1$, so then $x = 1$. For other values of y , we can divide through by $\log y$, to get

$$2 = y, \quad (5)$$

from which we get $x = 4$.

So

$$x = \{1, 4\}. \quad (6)$$