Math Diversion Problem 807

P. Reany

September 19, 2025

> The Axiom of Choice is obviously true; the Well Ordering
> Principle is obviously false; and who can tell
> about Zorn's Lemma?
> — Jerry Bona

```
Source: https://www.youtube.com/watch?v=0eWSViFOqFI
Title: Olympiads PROBLEM || n + 9 = n
 Presenter: Learn with Christian Ekpo
```

1 Problem

Given the relation

$$(-9)^n = 9\,, \tag{1}$$

find the complex solutions for n.

2 Solution

Let's simplify by replacing (1) by

$$(-1)^n 9^n = 9\,. \tag{2}$$

But $-1 = e^{i\pi}$, so

$$(e^{i\pi n})9^n = 9\,, \tag{3}$$

which we can write as

$$e^{i\pi n} = 9^{1-n}\,, \tag{4}$$

Time to take the natural logarithm:

$$i\pi n = (1-n)\ln 9\,. \tag{5}$$

Solving this for n, we have that

$$n = \frac{\ln 9}{i\pi + \ln 9}\,. \tag{6}$$