Math Diversion Problem 808

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Heuristic note: When it comes time to guess,
start by guessing simple.
— The Author

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Source:  The Ether of Great Mathematical Ideas
Title:  The Daily Commute
 Presenter: Patrick
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1 Problem

Let G be a group in which

$$g^2 = e,\ \forall\ g \in G\,. \tag{1}$$

Show that G is an abelian group, which means that for all $g, a \in G$,

$$ga = ag\,. \tag{2}$$

2 Solution

We begin by noting that in this group

$$g^{-1} = g\,. \tag{3}$$

We'll complete this problem's proof by contradiction. Assume there exists some $g, a \in G$ such that

$$ga \neq ag\,. \tag{4}$$

Then

$$gag^{-1}a^{-1} \neq e\,, \tag{5}$$

or rather

$$gaga \neq e\,. \tag{6}$$

And finally,

$$(ga)^2 \neq e\,. \tag{7}$$

But this cannot be because $ga \in G$ and hence

$$(ga)^2 = e \tag{8}$$

by assumption. Therefore, contradiction. Thus G must be abelian.