Math Diversion Problem 816

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> Great math books need to start with intuition,
> context, and simple examples, and from there
> build up to formalism and proofs.
> — Luca di Beo

1 Problem

Find the integral

$$I = \int a^x \, dx \, . \tag{1}$$

2 Solution

Now, it may not be obvious, but a way to proceed could be to rewrite (1) in the form

$$\int e^z \, dz = e^z + C \, . \tag{2}$$

So, let's try this approach. Set

$$e^y = a^x \, , \tag{3}$$

and solve for y by taking the natural log of both sides:

$$y = x \ln a \, , \tag{4}$$

Thus,

$$\begin{aligned} I = \int a^x \, dx = \int e^{x \ln a} \, dx = \frac{1}{\ln a} \int e^{x \ln a} \, d(\ln a \, x) \\ = \frac{1}{\ln a} e^{x \ln a} + C = \frac{1}{\ln a} a^x + C \, . \end{aligned} \tag{5}$$