Math Diversion Problem 817

P. Reany

September 28, 2025

It is a capital mistake to theorize in advance of the facts.
— Sherlock Holmes (Jeremy Brett)
[Episode *The Second Stain*]

1 Problem

A vendor wishes to buy fruit concentrate at \$10 per quart and mix it with water in two different proportions to make two different drinks, at two different sales prices, to make two different profits off each mix. The high concentrate mix is in ratio 1 : 3 concentrate to water, and the low concentrate mix is in ratio 1 : 4 concentrate to water. The vendor sells the high concentrate for \$6 per quart, and the low concentrate for \$4 per quart. If on a given evening, he sells twice as much of the low concentrate mix as high concentrate mix, what profit does the vendor make per quart?

2 Solution

Let's begin with a figure.

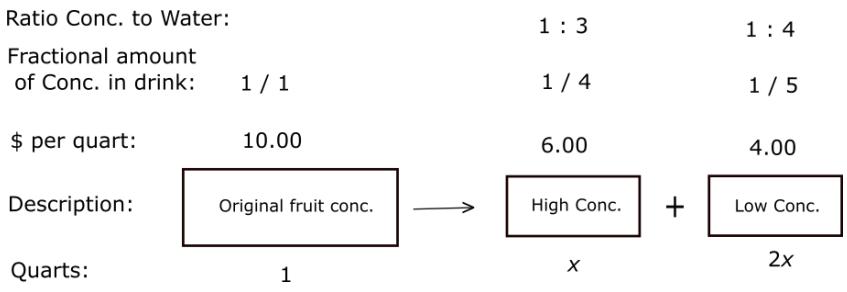

Figure 1. The overall volumes of the mixes are not conserved, but their fractional amounts in the mixes are.

Let's be clear on what we mean by Profit.

$$\text{Profit} = (\text{Sales Price}) - (\text{Cost})\,. \tag{1}$$

Profit per quart of fruit concentrate is no more complicated than to write:

$$\text{Profit/quart} = (\text{Sales Price/quart}) - (\text{Cost/quart})\,. \tag{2}$$

The cost/quart of fruit concentrate is simple. It's just the original \$10.00 because we assume no cost to add all the water we want to make the two mixes. The next equation we can write down is for the conservation of fruit concentrate (with the unit of quarts suppressed)[1]:

$$1 = (1/4)x + (1/5)(2x)\,. \tag{3}$$

which has solution $x = 20/13$ quarts for the sale of the high concentrate. Therefore $2x = 40/13$ quarts for the sales of the low concentrate.

$$\begin{aligned}(\text{Sales Price/quart}) &= 6.00x + 4.00(2x) \\ &= 6.00(20/13) + 4.00(40/13) \\ &\approx 21.54\,. \end{aligned} \tag{4}$$

which is the dollar amount per quart. So, take away the original cost of \$10 per quart, and we get the profit per quart to be \$11.54.

[1] Think of it this way. If I take a pile of nickels and divide it into two piles, I still have the same number of nickels, right? Now, if I add a bunch of pennies to each pile of nickels, I increase the number of coins per pile, but the overall number of nickels has remained the same. In like manner, adding water to each mix changes the volumes of each, but leaves invariant the original amount of fruit concentrate.