Math Diversion Problem 818

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September 29, 2025

The law is what we live with, Inspector.
Justice is sometimes harder to achieve.
— Sherlock Holmes (Jeremy Brett)
[Episode *The Red Circle*]

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Source: The Ether of Great Mathematical Ideas
 Title:  The tangent vector to a line in the plane
  Presenter:  Patrick
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1 Problem

Given the equation of a line L in the plane in standard form as

$$ax + by + c = 0\,, \tag{1}$$

find a vector $\mathbf{t}$ tangent to this line.[1]

2 Solution

Note: Let $\boldsymbol{\sigma}_1$ represent a unit vector in the $+x$ direction, and let $\boldsymbol{\sigma}_2$ represent a unit vector in the $+y$ direction.

How to get $\mathbf{t}$? This vector will be characterized by its slope — that is, its rise over its run, to use the old language; but otherwise its length doesn't matter.

The slope $\Delta y/\Delta x$ of the line in (1) is

$$\frac{\Delta y}{\Delta x} = \frac{-a}{b}\,. \tag{2}$$

Okay, so a vector that has the same rise-over-run as the line could be

$$\mathbf{t} = \Delta x\,\boldsymbol{\sigma}_1 + \Delta y\,\boldsymbol{\sigma}_2 = b\boldsymbol{\sigma}_1 - a\boldsymbol{\sigma}_2\,. \tag{3}$$

If we want it as a unit vector, we could write

$$\hat{\mathbf{t}} = (b\boldsymbol{\sigma}_1 - a\boldsymbol{\sigma}_2)/\sqrt{a^2 + b^2}\,. \tag{4}$$

[1] We assume that $b \neq 0$. If it is, the slope is infinite, so we launch an exception and set $\mathbf{t} = \pm\boldsymbol{\sigma}_2$.