Math Diversion Problem 824

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You have to know what to look for, so you can spot it.
— Papago Indian drug-enforcement
border scout

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Source: Conceptual Mathematics: a first introduction
      to categories, 2nd Ed
Title: Isomorphims, Exercise 3b
  Presenter:  F. W. Lawvere & S. H. Schanuel
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1 Problem 3b

Function $f : A \to B$ has an inverse. Show that if

$$h \circ f = k \circ f \,, \tag{1}$$

then

$$h = k \,. \tag{2}$$

2 Solution

Let's begin by drawing a figure, where A, B, C are sets.

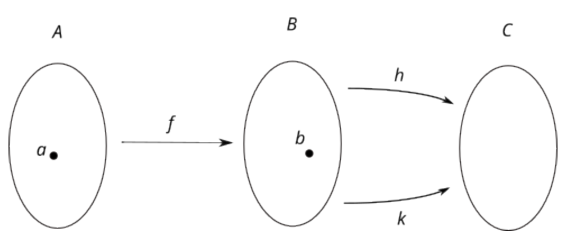

Figure 1. f^{-1} exists and $h \circ f = k \circ f$, where $h, k : B \to C$.

What does it mean that $h = k$? It means that for every $b \in B$

$$h(b) = k(b)\,. \tag{3}$$

We note first that since f has an inverse then f is surjective. So what do we have to work with? From (1), we have that

$$(h \circ f)(a) = (k \circ f)(a) \quad \forall\ a \in A\,, \tag{4}$$

which means that

$$h(f(a)) = k(f(a)) \quad \forall\ a \in A\,. \tag{5}$$

Somehow we have to work in the fact that f has an inverse. So $f(a) = b$. And therefore $f^{-1}(b) = a$. Substituting this into (5), we have that

$$h(f(f^{-1}(b))) = k(f(f^{-1}(b))) \quad \forall\ b \in B\,. \tag{6}$$

On canceling ff^{-1}, we get

$$h(b) = k(b) \quad \forall\ b \in B\,. \tag{7}$$

Hence

$$h = k\,. \tag{8}$$