Math Diversion Problem 828

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> Solving math puzzles can enhance cognitive functions such
> as problem-solving, logical reasoning, and memory,
> contributing to overall brain health.
> — Daniel Levitin (renowned neuroscientist)

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Source: The Ether of Great Mathematical Ideas
Title: A word problem, model with a time line
  Presenter:  Patrick
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1 Problem

Twenty workers can do a job in 35 days. Beginning with 20 workers, work proceeds as normal for 11 days. After that time, 5 workers quit but are not replaced for the next 4 days. How many workers should be hired on day sixteen so that the job will be finished on the 35th day?

2 Solution

Let's visualize this as a timeline figure.

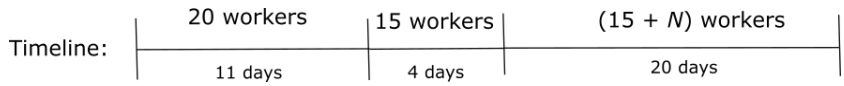

Figure 1. We divide up the whole job into three time periods, each contributing to the whole job.

We'll make the assumption right off that the rate R at which each worker works (in job/day) is the same for all of them. We can easily calculate R from the first information given:

$$R(35 \text{ days}) + R(35 \text{ days}) + \cdots + R(35 \text{ days}) = 1[\text{job}]\,, \tag{1}$$

where there are 20 terms in this sum. Therfore, we get

$$(20R)(35 \text{ days}) = 1[\text{job}]\,, \tag{2}$$

from which we have that $R = \frac{1}{700}$ job/day.

Next, we divide up the whole job into three contributing parts and sum on those three contributions:

$$\begin{aligned} 1 \text{ job} &= (\text{PJDB 1st part}) + (\text{PJDB 2nd part}) + (\text{PJDB 3rd part}) \\ &= \left(20\,\frac{1}{700}\right)(11) + \left(15\,\frac{1}{700}\right)(4) + \left((15+N)\,\frac{1}{700}\right)(20)\,. \end{aligned} \tag{3}$$

Multiplying through by 700, we get

$$700 = (20)11 + (15)4 + (15+N)20\,, \tag{4}$$

which has solution $N = 6$. Meaning, that for the last twenty days of work, six more workers should be hired to finish the job on the 35th day.