Math Diversion Problem 829

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October 8, 2025

Truth, like oil, will in time rise to surface.
— Charlie Chan

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Source: https://www.youtube.com/watch?v
             =2iBNo4j3vRo&list=PL3E4136E122545FBE
Title: Gamma Function - Part 1 - Functional Equation
  Presenter:  MrYouMath
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1 Introduction

This is the first part of a 12 -part series on the Gamma function. What I'm presenting here is what I refer to as the 'read-a-long notes' to the videos. They are brief on explanations. For better explanations, please see the videos by MrYouMath, as listed above.

2 The Gamma Function – Part 1

Let s be a complex variable. Then the Gamma function of s is given as

$$\Gamma(s) \equiv \int_0^\infty t^{s-1} e^{-t} dt \quad \text{where } \mathrm{Re}(s) > 0\,. \tag{1}$$

Something profound is the result of merely letting $s \to s+1$ and then integrating by parts:

$$\begin{aligned}
\Gamma(s+1) &= \int_0^\infty t^s e^{-t} dt \\
&= -e^{-t} t^s \big|_0^\infty - \int_0^\infty \frac{dt^s}{dt} \left(-e^{-t}\right) dt \\
&= \int_0^\infty s t^{s-1} e^{-t} dt \\
&= s \int_0^\infty t^{s-1} e^{-t} dt \\
&= s\Gamma(s)\,.
\end{aligned} \tag{2}$$

Hence, for n a positive integer:

$$\Gamma(n+1) = n!\,. \tag{3}$$