Math Diversion Problem 834

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> A new scientific truth does not triumph by convincing its
> opponents and making them see the light, but rather
> because its opponents die and a new generation
> grows up that is familiar with it.
> — Max Planck

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Source: The Ether of Great Mathematical Ideas
Title: An Integral
  Presenter:  Patrick
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1 Problem

Find the integral

$$I = \int \frac{9^x - 4^x}{3^x + 2^x}\, dx\,. \tag{1}$$

2 Lemma

$$\int a^x\, dx = \frac{1}{\ln a} a^x + C\,. \tag{2}$$

3 Solution

We will use the previous result, but to do so, we must first recognize that $9 = 3^2$ and $4 = 2^2$ and then factor the numerator by using the formula for the difference

of two squares. Thus,

$$\begin{aligned} I &= \int \frac{3^{2x} - 2^{2x}}{3^x + 2^x}\, dx \\ &= \int \frac{(3^x - 2^x)(3^x + 2^x)}{3^x + 2^x}\, dx \\ &= \int 3^x - 2^x\, dx \\ &= \frac{1}{\ln 3} 3^x - \frac{1}{\ln 2} 2^x + C\,. \end{aligned} \tag{3}$$