# Math Diversion Problem 835

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Source: https://www.youtube.com/watch?v
             =2iBNo4j3vRo&list=PL3E4136E122545FBE
Title: Gamma Function - Part 4 - Gamma and sine functions together
  Presenter:  MrYouMath
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## 1 Introduction

This is the fourth part of a 12-part series on the Gamma function. What I'm presenting here is what I refer to as the 'read-a-long notes' to the videos. They are brief on explanations. For better explanations, please see the videos by MrYouMath, as listed above.

## 2 Gamma and sine functions together

Starting with

$$\Gamma(s) = \lim_{n\to\infty}\left[\frac{n^s}{s}\prod_{i=1}^{n}\frac{i}{(s+i)}\right] = \lim_{n\to\infty}\left[\frac{n^s}{s}\prod_{i=1}^{n}\frac{1}{(1+s/i)}\right], \tag{1}$$

and replacing $s$ by its negative, we have that

$$\Gamma(-s) = \lim_{n\to\infty}\left[\frac{n^{-s}}{-s}\prod_{i=1}^{n}\frac{1}{(1-s/i)}\right]. \tag{2}$$

On multiplying these together and rearranging, we have that

$$\Gamma(s)\Gamma(-s)(-s) = \lim_{n\to\infty}\left[\frac{1}{s}\prod_{i=1}^{n}\frac{1}{(1-s^2/i^2)}\right]. \tag{3}$$

This gives us

$$\Gamma(s)\Gamma(1-s) = \frac{1}{s}\prod_{i=1}^{n}\frac{1}{(1-s^2/i^2)}. \tag{4}$$

And, even simpler,[1]

$$\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin\pi s}. \tag{5}$$

[1] See Lecture 7 of the Riemann Zeta Function, or refer to the Read-Along notes.