Math Diversion Problem 836

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Zathras is used to being beast of burden to other
people's needs. Very sad life... Probably
have very sad death. But, at least
there is symmetry.
—Zathras (a character on Babylon 5)

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Source: https://www.youtube.com/watch?v=zUBKDZQy0p8
Title: The Equation That Connects Everything | P582
  Presenter:  aplusbi
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1 Problem

Given the relation

$$\cos z - i \sin z = \frac{1}{e}\,, \tag{1}$$

solve for z.

2 Solution

The Given relation can be rewritten as

$$e^{-iz} = e^{-1} = e^{-1+2\pi i n} \quad \text{for} \quad n \in \mathbb{Z}\,. \tag{2}$$

Next, take the natural logarithm:

$$-iz = -1 + 2\pi i n \quad \text{for} \quad n \in \mathbb{Z}\,. \tag{3}$$

Now, multiply through by i:

$$z = -i - 2\pi n \quad \text{for} \quad n \in \mathbb{Z}\,. \tag{4}$$

But we can finalize this down to

$$z = 2\pi n - i \quad \text{for} \quad n \in \mathbb{Z}\,, \tag{5}$$

because n will take on every integer value, so the sign in front of the real term doesn't matter.