Math Diversion Problem 843

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I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

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Source: https://www.youtube.com/watch?v
             =2iBNo4j3vRo&list=PL3E4136E122545FBE
Title: Gamma Function - Part 8 - The Euler Integral II
  Presenter:  MrYouMath
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1 Introduction

This is the eighth part of a 12-part series on the Gamma function. What I'm presenting here is what I refer to as the 'read-a-long notes' to the videos. They are brief on explanations. For better explanations, please see the videos by MrYouMath, as listed above.

2 The Euler Integral II (the Sinc function) – Part 8

The sinc function is given as $\operatorname{sinc} x = (\sin x)/x$. Its integral is

$$\int_0^\infty \frac{\sin x}{x}\,dx = \frac{\pi}{2}\,. \tag{1}$$

Using the final equation of the last section, and setting

$$a = 0\,,\quad b = 1\,,\quad n = 1\,,\quad s = 0\quad \text{(with effect taken in the limit)}\,, \tag{2}$$

and noting that $|\,p\,| = \sqrt{a^2+b^2} = 1$, and $\alpha = \pi/2$, then from

$$\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin \pi s}\,, \tag{3}$$

we get

$$\sin(\pi s/2)\cdot\frac{\pi}{\sin\pi s\,\Gamma(1-s)} = \int_0^\infty (u^{s-1}\sin u)\,du\,. \tag{4}$$

With slight modification, we get

$$\frac{\pi}{2}\left[\frac{\sin\left(\pi s/2\right)}{\pi s/2}\,\frac{\pi s}{\sin \pi s\,\Gamma(1-s)}\right]=\int_0^\infty \left(u^{s-1}\sin u\right)du\,. \tag{5}$$

Next, we use that $\lim_{x\to 0}\dfrac{\sin x}{x}=1$, so

$$\lim_{s\to 0}\left[\frac{\sin\left(\pi s/2\right)}{\pi s/2}\,\frac{\pi s}{\sin \pi s\,\Gamma(1-s)}\right]=\frac{1}{\Gamma(1)}=1\,. \tag{6}$$

Then,

$$\int_0^\infty \frac{\sin u}{u}\,du=\frac{\pi}{2}\,. \tag{7}$$