

# Math Diversion Problem 844

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You either get control of your lusts and feelings of  
entitlement, or they will get control of you.

— The Author

Source: The Ether of Great Mathematical Ideas

Title: The Angle Bisector Theorem

Presenter: Patrick

## 1 Solution

**Definition:** Two triangles are said to be *similar* if their corresponding angles are congruent (have the same measures).

**Lemma:** The corresponding sides of similar triangles are in the same proportion.

The Angle Bisector Theorem is ancient and has many proofs. Let's consider a (possibly) new proof here. Refer to the triangle  $ABC$  in Fig. 1.

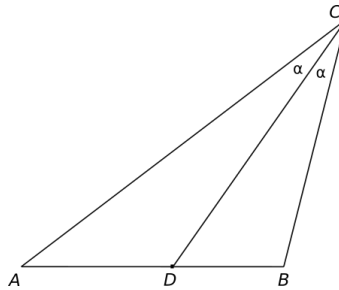


Figure 1. The triangle defined by the vertices  $A, B, C$ , has a point  $D$  placed on line segment  $\overline{AB}$  so as to bisect the angle at vertex  $C$ .

The triangle defined by points  $A, B, C$  is an arbitrary triangle. The line segment  $\overline{CD}$  bisects the angle at vertex  $C$ . Then, the point  $D$  divides the line segment  $\overline{AB}$  as per the following proportion:  $\frac{AC}{AD} = \frac{BC}{BD}$ .

## 2 Step One

Construct a circle of radius  $CB$  and centered at  $C$ , as in Fig. 2. The point  $F$  is the meet of the extension of line segment  $\overline{AC}$  to the circle. Point  $E$  is the meet of the circle with line segment  $\overline{AC}$ .

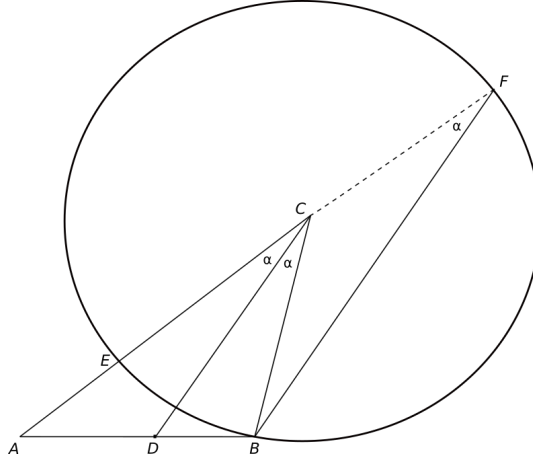


Figure 2. After constructing the circle centered at  $C$  of radius  $CB$ , we use the Inscribed Angle Theorem to show that the angle at  $F$  is half the central angle of  $ECB$  (i.e.,  $2\alpha$ ), thus is equal to  $\alpha$ .

## 3 Step Two

There is a theorem from geometry that if two lines cut a third line at equal angles, the two lines are parallel. From the definition above, since the corresponding angles of the two triangles  $FAB$  and  $CAD$  are congruent (the reader may prove this), the triangles are similar. Thus, by the Lemma above, the corresponding parts of the two triangles are proportional, therefore,

$$\frac{AC}{AD} = \frac{AF}{AB} = \frac{AC + CF}{AB} = \frac{AC + CB}{AB} = \frac{AC}{AB} + \frac{CB}{AB}. \quad (1)$$

On subtracting  $\frac{AC}{AB}$  from both sides, we get,

$$AC \left[ \frac{1}{AD} - \frac{1}{AB} \right] = \frac{CB}{AB}. \quad (2)$$

Using that  $AB - AD = DB$  and applying some algebra, we get that

$$\frac{AC}{AD} = \frac{BC}{BD}. \quad (3)$$