Math Diversion Problem 845

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You cannot ask us to take sides against arithmetic.
— attributed to Winston Churchill

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Source: https://www.youtube.com/watch?v
             =2iBNo4j3vRo&list=PL3E4136E122545FBE
Title: Gamma Function - Part 9 -
             Euler Integral III (Fresnel Integrals)
  Presenter:  MrYouMath
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1 Introduction

This is the ninth part of a 12-part series on the Gamma function. What I'm presenting here is what I refer to as the 'read-a-long notes' to the videos. They are brief on explanations. For better explanations, please see the videos by MrYouMath, as listed above.

2 Preparation

$$\frac{\Gamma(s)}{n\,|\,p\,|^s}\begin{bmatrix}\cos(\alpha s)\\ \sin(\alpha s)\end{bmatrix} = \int_0^\infty u^{ns-1}e^{-au^n}\begin{bmatrix}\cos(bu^n)\\ \sin(bu^n)\end{bmatrix}du\,. \tag{1}$$

3 The Euler Integral III (Fresnel Integrals) – Part 9

The Fresnel Integrals have the forms

$$\begin{aligned}\int_0^\infty \sin u^2\,du &= \frac{\sqrt{2\pi}}{4}\,,\\ \int_0^\infty \cos u^2\,du &= \frac{\sqrt{2\pi}}{4}\,.\end{aligned} \tag{2}$$

Let's bring out (1)

$$\frac{\Gamma(s)}{n\,|\,p\,|^s}\sin(\alpha s) = \int_0^\infty u^{ns-1}e^{i\alpha u^n}\sin(bu^n)\,du\,. \tag{3}$$

But we have need of specific values:

$$s = \tfrac{1}{2}\,, \quad \Gamma(\tfrac{1}{2}) = \sqrt{\pi}\,, \quad a = 0\,, \quad b = 1\,, n = 2\,, \quad \alpha = \pi/2\,. \tag{4}$$

On substituting in, we get

$$\tfrac{1}{2}\Gamma(\tfrac{1}{2}) \sin \pi/4 = \int_0^\infty \sin u^2 \, du\,. \tag{5}$$

Or

$$\int_0^\infty \sin u^2 \, du = \frac{\sqrt{2\pi}}{4}\,, \tag{6}$$

and similarly for the cosine case.