Math Diversion Problem 860

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A man convinced against his will is unconvinced still.
— Proverb

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Source: https://www.youtube.com/watch?v=z9pzZ1qjMXA
Title: This Expression Breaks Reality! | P585
  Presenter:  aplusbi
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1 Problem

Given the expression

$$\phi \equiv i^{i^{i^{i}}} \,, \tag{1}$$

express ϕ in a more conventional form.

2 Solution

First, a simplification. Let

$$\alpha \equiv i^i = (e^{i\pi/2})^i = e^{-\pi/2} \,, \tag{2}$$

which is a real number.

Hence,

$$\phi = i^{i^{\alpha}} \,. \tag{3}$$

Let's take a natural logarithm across this, to get

$$\begin{aligned} \ln\phi = i^{\alpha} \ln i &= i^{\alpha} \ln e^{i\pi/2} \\ &= (e^{i\pi/2})^{\alpha}(i\pi/2) \\ &= (\pi/2)(e^{i\alpha\pi/2})e^{i\pi/2} \\ &= (\pi/2)(e^{i(\alpha+1)\pi/2}) \,. \end{aligned} \tag{4}$$

Finally,

$$\phi = e^{\pi/2} \exp\left[\, e^{i(\alpha+1)\pi/2} \,\right] . \tag{5}$$