Math Diversion Problem 864

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CONCERNING HORSES

I've been through the desert
On a horse with no name
It felt good to be out of the rain
In the desert, you can remember your name
'Cause there ain't no one for to give you no pain.
— 'A Horse with No Name' (*America*, 1971)

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Source: https://www.youtube.com/watch?v=4EeKtTY1GwI
Title: A Polynomial Riddle | Can You Solve?
  Presenter:  SyberMath
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1 Problem

Given the relation, where $P(x)$ is a polynomial,

$$P(x) - P'(x) = x^2 - 2\,, \tag{1}$$

solve for $P(x)$.

2 Solution

Since we are told that $P(x)$ is a polynomial, we know that it has the form of a sum of terms as powers of x. But what's the highest power of x that works in (1)? The highest power we can use i 2; if we go any higher, we'll have those higher term on the LHS with no corresponding terms on the RHS.

We'll know $P(x)$ when we know its coefficients. We could just guess, but I prefer to use the method of undetermined coefficients. So, we're assuming that $P(x)$ is a quadratic with coefficients, say, a, b, c:

$$P(x) = ax^2 + bx + c\,. \tag{2}$$

Therefore,

$$P'(x) = 2ax + b\,. \tag{3}$$

Now, on substituting (2) and (3) into (1), we have that

$$(ax^2 + bx + c) - (2ax + b) = x^2 - 2\,. \tag{4}$$

The coefficients on the RHS and LHS of this equation have to match according to respective powers of x, that being 2,1,0; so, matching in that order, we get

$$\begin{aligned} a &= 1\,, \\ b - 2a &= 0\,, \\ c - b &= -2\,. \end{aligned}$$

Immediately, we see that $a = 1$, so $b = 2$, and so $c = -2 + 2 = 0$. This mean that

$$P(x) = x^2 + 2x\,. \tag{5}$$