Math Diversion Problem 866

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CONCERNING HORSES

> Hast thou given the horse strength? hast thou clothed his neck with thunder? Canst thou make him afraid as a grasshopper? the glory of his nostrils is terrible. He paweth in the valley, and rejoiceth in his strength: he goeth on to meet the armed men. He mocketh at fear, and is not affrighted; neither turneth he back from the sword.
> — Job 39:19–22

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Source: The Ether of Great Mathematical Ideas
Title: It can get confusing
  Presenter:  Patrick
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1 Problem

Nine liters are drawn from a tank full of wine. Then 9 liters of pure water are added to the tank and the mix is allowed to homogenize. After that, 9 more liters are drawn off and again replaced by 9 liters of pure water and allowed to homogenize. If the final wine-to-water mix is in ratio 16:9, how much does the full tank hold?

2 Solution

We can solve this problem in two steps. First, let x be the full volume of the tank. Second, after replacing 9 liters of the original wine by 9 liters of water, we end up with Mix 1 with wine-to-water ratio $(x - 9) : 9$ and fraction of wine-to-total mixture $(x - 9)/x$ (no figure for this point in the analysis).

Next, we draw off 9 more liters of this mixture and that takes us to Step 2, the setup in Figure 2.

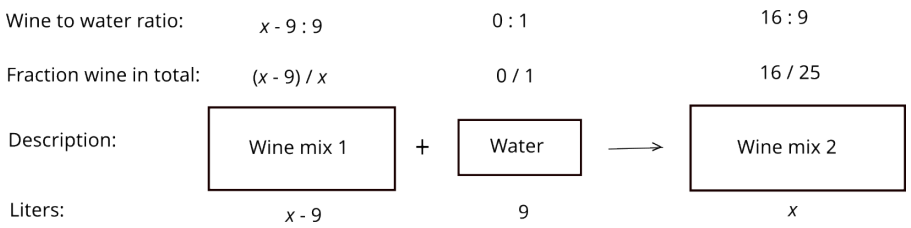

Figure 1. Two-step analysis will do the trick.

The process of adding water to Mixture 1 in Figure 2 tells us that the total amount of wine is conserved in this process:

$$\frac{x-9}{x}\,(x-9) + 0(9) = \frac{16}{25}\,(x)\,, \tag{1}$$

which has solution $x = 45$ liters.[1]

[1] The possible root $x = 5$ is unphysical because it does not satisfy the constraint $x - 9 > 0$.