Math Diversion Problem 879

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No mystery is closed to an open mind.
—Tim White
Sightings TV show

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Source:  https://answers.yahoo.com/question/index?qid
                        =20081230083424AA2lfMV
Title: Hard stoichiometry problem
  Presenter:  Yahoo Answer site
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1 Problem: A Two-Step Dehydration Process

PROBLEM:

Hard stoichiometry problem?
Can some show me how to do this?

You are given a mixture of three hydrated salts:

$$\mathrm{Na_2CO_3 \cdot 10H_2O}, \quad \mathrm{MgSO_4 \cdot 7H_2O}, \quad \text{and} \quad \mathrm{CuSO_4 \cdot 5H_2O}\,.$$

The total mass of the mixture is 12.123 grams. When the mixture is heated gently, the following two reactions occur:

$$\mathrm{Na_2CO_3 \cdot 10H_2O(s)} \longrightarrow \mathrm{Na_2CO_3 \cdot 7H_2O(s)} + \mathrm{3H_2O(g)} \tag{1a}$$

$$\mathrm{MgSO_4 \cdot 7H_2O(s)} \longrightarrow \mathrm{MgSO_4 \cdot H_2O(s)} + \mathrm{6H_2O(g)} \tag{1b}$$

After these reactions are complete, the mass of the mixture has decreased to 9.049 grams. This mixture is then heated more strongly, and the following additional reactions occur:

$$\mathrm{Na_2CO_3 \cdot 7H_2O(s)} \longrightarrow \mathrm{Na_2CO_3(s)} + \mathrm{7H_2O(g)} \tag{2a}$$

$$\mathrm{MgSO_4 \cdot H_2O(s)} \longrightarrow \mathrm{MgSO_4(s)} + \mathrm{H_2O(g)} \tag{2b}$$

$$\mathrm{CuSO_4 \cdot 5H_2O(s)} \longrightarrow \mathrm{CuSO_4(s)} + \mathrm{5H_2O(g)} \tag{2c}$$

After this final heating, the mass of the mixture has decreased to 6.412 grams. From this information, calculate the masses of each of the three compounds in the original mixture.

The provided answers are:

mass of $Na_2CO_3 \cdot 10H_2O$	1.374 g
mass of $MgSO_4 \cdot 7H_2O$	6.418 g
mass of $CuSO_4 \cdot 5H_2O$	4.331 g

2 Solution

We first note that all the chemical equations displayed above are balanced. Next, we consider the paired equations in the first reaction, given by (1a) and (1b).

Let's set the initial values of moles of $Na_2CO_3 \cdot 10H_2O$, $MgSO_4 \cdot 7H_2O$, and $CuSO_4 \cdot 5H_2O$ to a, b, c, respectively. Once we know their values, we can calculate the masses of their respective hydrates quite easily.

Before we make stoich diagrams to help out, we should first make some simplifying notations for the three hydrates. For each generic hydrate Hy, the symbol Hy_n represents $Hy \cdot n\,H_2O$, and Hy_0 represents the anhydrous salt. Further, letting $X \sim Na_2CO_3$, $Y \sim MgSO_4$, and $Z \sim CuSO_4$, then, for examples

$$X_{10} \sim Na_2CO_3 \cdot 10H_2O,$$

$$Y_1 \sim MgSO_4 \cdot H_2O,$$

$$Z_0 \sim CuSO_4.$$

Now for a diagram depicting the first heating event and its consequences:

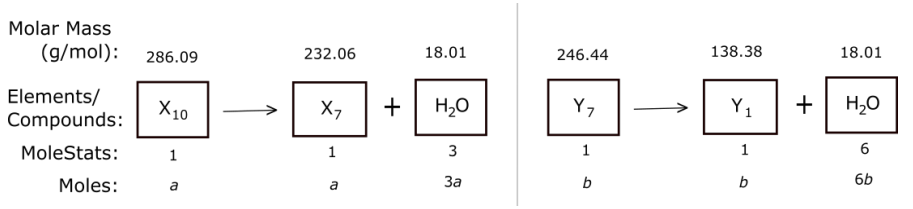

Figure 1a. Diagram of the two independent reactions. Here we've introduced the variables a and b to represent, respectively, the moles of X_{10} and Y_7. Once we know their values we can easily calculate their corresponding masses.

In Figure 3a, adopting the simplifying notations above, we represent the first two independent reactions that occurred in the first heating event. From the given information, we can calculate the mass of the evolved water gas. Second, the mole counts for the products X_7 and Y_1 will be passed along to the initial reactant mole counts during the second heating event.

One thing we know for sure from the first reaction is the mass of the evolved water gas, being the difference between the original mass of the mixture and

the residue mass of the mixture after the first heating event:

$$\text{Mass } H_2O \text{ evolved in first reaction} = 12.123\,\text{g} - 9.049\,\text{g} = 3.074\,\text{g}\,, \tag{3}$$

which we can then write in term of its moles and the molar mass of water, $18.01\,\text{g}{\cdot}\text{mol}^{-1}$:

$$(3a + 6b)\text{mol}\,(18.01\,\text{g}{\cdot}\text{mol}^{-1}) = 3.074\,\text{g}\,, \tag{4}$$

where we used the fact that the total moles of water produced is the sum of the moles from the left reaction (being $3a$) and the moles from the right reaction (being $6b$). We can rewrite this last equation for moles more simply as

$$3a + 6b = 0.17068\,. \tag{5}$$

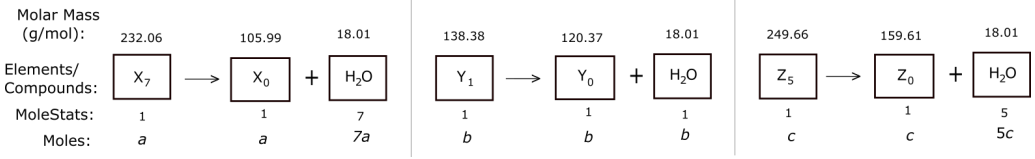

Figure 1b. This figure represents the second heating event.

Noting that Eq. (5) provides us with one equation to solve for the triple variables a, b, c, we need only two more equations, derived from the information in Figure 3b, to form a set of three equations in three unknowns, which we will then solve simultaneously.

First, we know the mass of the final anhydrous salts to be 6.412 grams. Thus (Total is the sum of its parts)

$$\text{Mass of } X_0 + \text{Mass of } Y_0 + \text{Mass of } Z_0 = 6.412\,, \tag{6}$$

which can be expressed in terms of their moles and molar masses as

$$105.99a + 120.37b + 159.61c = 6.412\,. \tag{7}$$

And, arguing similarly to how we derived Eq. (5) for the mass of the water gas evolved in the first heating event, we have for the second heating event

$$(7a + b + 5c)\text{mol}\,(18.01\,\text{g}{\cdot}\text{mol}^{-1}) = 2.637\,\text{g}\,, \tag{8}$$

which simplifies to

$$7a + b + 5c = 0.146419\,. \tag{9}$$

So, on solving (5), (7), and (9) simultaneously for a, b, c, Wolframalpha gives

moles of $Na_2CO_3 \cdot 10H_2O$	$a = 0.00481197$
moles of $MgSO_4 \cdot 7H_2O$	$b = 0.0260407$
moles of $CuSO_4 \cdot 5H_2O$	$c = 0.0173389$

Then, converting these mole values to gram values, we get

mass of $Na_2CO_3 \cdot 10H_2O$	1.377 g
mass of $MgSO_4 \cdot 7H_2O$	6.417 g
mass of $CuSO_4 \cdot 5H_2O$	4.329 g

3 Appendix: How to interpret the Stoich diagrams

There are four main types of data in the stoich diagrams I make. The most common are data from given information, from the coefficients of the balanced equation, and from data tables, such as a periodic table of elements for molar mass information. This kind of data I do not mark up. The second kind of data in stoich diagrams comes from computations based on data in the same column, for which I use the turnstile ($\vdash$) to indicate. The third kind of data is a result in one column that required data from at least one other column to calculate it, and I indicate that kind of value or result by use of the underlining. The fourth kind of data in the figures is the result of combining given information to derive a secondary value. I indicate this kind of data with a right arrowhead ($\blacktriangleright$).