Math Diversion Problem 884

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If one has misjudged the true nature of the problems one faces, one will likely misjudge the correct solutions to the problems, making things worse.
— The Author

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Source:  The Ether of Great Mathematical Ideas
Title: An Induction Problem
 Presenter:  Patrick
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1 Problem

Using induction, show that

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \,. \tag{1}$$

2 Solution

To do this problem by induction, we need to accomplish three things. The first is to show that a suitable base case works. For my base case, I choose $n = 1$. Then

$$\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{1(1+1)} = \frac{1}{2} = \frac{1}{1+1} \,. \tag{2}$$

That works.

Next, we formulate an inductive hypothesis, which we assume is true. It will be

$$\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1} \,. \tag{3}$$

Lastly, given the preceding, we need to show that

$$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \frac{k+1}{k+2} \,. \tag{4}$$

Thus,

$$\begin{aligned}
\sum_{i=1}^{k+1} \frac{1}{i(i+1)} &= \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{(k+1)(k+2)} \\
&= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} \quad \text{[using (3)]} \\
&= \frac{k(k+2)+1}{(k+1)(k+2)} \\
&= \frac{k^2+2k+1}{(k+1)(k+2)} \\
&= \frac{(k+1)^2}{(k+1)(k+2)} \\
&= \frac{k+1}{k+2}\,, \qquad (5)
\end{aligned}$$

and thus we have established (4).