Math Diversion Problem 892

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> Mathematics is the art of reducing any
> problem to linear algebra.
> — William Stein

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Source:  https://www.youtube.com/watch?v=8cp_ij0APW4
Title: Everyone Misses This Simple Algebra Trick!
 Presenter:  Mental Math
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1 Problem

Given the relations

$$x + y = 1\,, \tag{1a}$$

and

$$x^3 + y^3 = 7\,, \tag{1b}$$

find the value of

$$\phi = xy\,. \tag{2}$$

2 Solution

On cubing both sides of (1a), we get

$$x^3 + 3x^2y + 3xy^2 + y^3 = 1\,, \tag{3}$$

which be rewritten as

$$7 + 3xy(x + y) = 1\,. \tag{4}$$

But from (1a) and (2), we have that

$$7 + 3\phi = 1\,. \tag{5}$$

Solving this for ϕ, we get

$$\phi = -2\,. \tag{6}$$