Math Diversion Problem 896

P. Reany

November 11, 2025

He that answereth a matter before he heareth it,
it is folly and shame unto him.
— Proverbs 18:13

The YouTube video is found at:

```
Source: https://www.youtube.com/watch?v=wgd6Gp-Kfbk
Title: This Question Tricked Thousands of Students
 Presenter: J Educational Tutorials
```

1 The Problem

Given the relation

$$x^2 - 5x + 7 = 0\,, \tag{1}$$

find the value of ϕ given by

$$\phi \equiv (x-2)^{90} + (3-x)^{90}\,. \tag{2}$$

2 The Solution

My first job is to rewrite ϕ as

$$\phi = (x-2)^{90} + (x-3)^{90}\,. \tag{3}$$

So, I chose as my 'first unipode' and follow-up:

$$a = (x-2)u_+ + (x-3)u_- \tag{4a}$$

$$= \frac{1}{2}\big[(x-2)+(x-3)\big] + \frac{1}{2}\big[(x-2)-(x-3)\big]\,u \tag{4b}$$

$$= \big(x - \frac{5}{2}\big) + \frac{1}{2}u\,. \tag{4c}$$

$$a^{90} = (x-2)^{90}u_+ + (x-3)^{90}u_- \tag{4d}$$

$$= \frac{1}{2}[(x-2)^{90} + (x-3)^{90}] + \frac{1}{2}[(x-2)^{90} - (x-3)^{90}]u \tag{4e}$$

$$= \frac{1}{2}\phi + \frac{1}{2}\psi u\,, \tag{4f}$$

where we defined

$$\psi = (x-2)^{90} - (x-3)^{90}\,. \tag{5}$$

Now, if we can find a numerical value for $x - 5/2$ in (4c), we can reduce a to a pure unipode. If we can complete the square on (1), we can solve for $(x-5/2)^2$

$$(x-5/2)^2 = -\frac{3}{4}\,. \tag{6}$$

On taking the square root on both sides, we have that

$$x - 5/2 = \pm i\frac{\sqrt{3}}{2}\,. \tag{7}$$

Therefore

$$a = \pm i\frac{\sqrt{3}}{2} + \frac{1}{2}u\,. \tag{8}$$

Now for a little algebraic sleight of hand, multiply through by u and reorder terms on the right:

$$au = \frac{1}{2} \pm I\frac{\sqrt{3}}{2} = e^{\pm I\pi/3}\,, \tag{9}$$

where we have introduced I as

$$I \equiv ui\,, \tag{10}$$

and

$$I^2 = (ui)^2 = -1\,. \tag{11}$$

In other words, I acts just like a unit imaginary.

So,

$$a^{90} = (au)^{90} = \left(e^{\pm I\pi}\right)^{30} = (-1)^{30} = 1\,. \tag{12}$$

On comparing this last result with (4f), we conclude that

$$\phi = 2\,. \tag{13}$$