Math Diversion Problem 898

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> If people knew how hard I worked to achieve my mastery,
> it wouldn't seem so wonderful after all.
> — Michelangelo

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Source: The Ether of Great Mathematical Ideas
Title: What is really meant by 2% milk?
 Presenter: Patrick
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1 The Problem

This is a word problem I made up, but probably someone else did also: I have long been curious what is really meant by 2% milk. According to one online source, it means that the milk fat content of 2% milk is 2% by weight. According to another source, whole milk is approximately 3.5% milk fat. Therefore, whole milk is about 96.5% skim milk. A few years ago I read on a 2% milk label that the milk inside the container was 35% less fat than whole milk. So, given just the information on that 2% milk label (and assuming it's true), what is the fat content of whole milk?

Right off, I'm going to make a possibly controversial simplifying assumption, which I hope will not introduce a big inaccuracy into the solution, by treating percentages by weight as percentages by volume.

2 Solution

This bring me to one of the tricks I have learned to employ in my Scheme: If you're given an unfamiliar situation, try to refashion it into a familiar pattern. Now, let's conceptualize how 2% milk is made. Put simply, starting, say, with 1 gallon of whole milk, we remove a certain amount of milk fat (y gallons) until what is left is 2% milk (x gallons). If we model this as a before-and-after process, we'll have

$$(\text{whole milk}) - (\text{some milk fat}) \to (2\%\ \text{milk})\,. \tag{1}$$

But I'd prefer to visualize this process going the other way, reconstituting the whole milk, to allow me to represent the problem visually in a very familiar way,

depicted in Figure 1. By the way, we lose no generality by choosing the whole milk to have a volume of 1 gallon.

In Figure 1, we see the constitutive relation between y and a, which was given to us by the claim that 2% milk is 35% less fat than whole milk. Now, our symbol for the percentage of milk fat in whole milk is a. Therefore, the amount of milk fat in one gallon of whole milk is

$$\big(\frac{a}{100}\frac{\text{gallons milk fat}}{\text{gallon whole milk}}\big)(1 \text{ gallon whole milk}) = \frac{a}{100}[\text{gallon whole milk}]\,. \quad (2)$$

But we are told that y is 35% of that, from which we get the constitutive equation

$$y = (0.35)\big(\frac{a}{100}\big)\,. \quad (3)$$

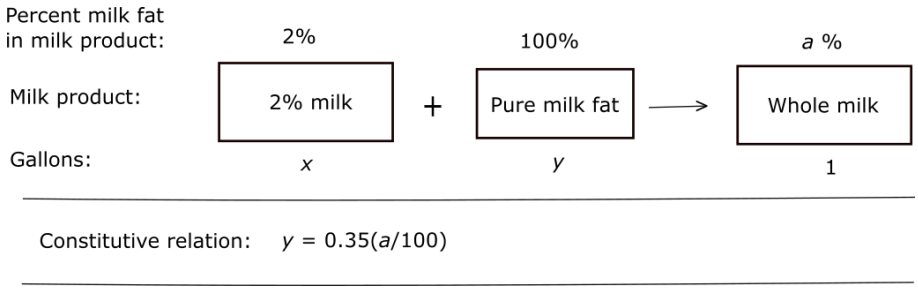

Figure 1. We can solve for the unknown value a% by reconstructing the original whole milk by putting back into it the milk fat we removed to produce 2% milk.

We can now write down the total is the sum of its parts, both before and after mixing component milk products. Balancing on milk fat, we get

$$0.02(x) + 1.00(y) = \frac{a}{100}(1)\,. \quad (4a)$$

Including the conservation of overall volumes, we get

$$x + y = 1\,. \quad (4b)$$

The solution to (3), (4a), and (4b) is (according to Wolframalpha.com)

$$a \approx 3.04414, \qquad x \approx 0.989346, \qquad y \approx 0.0106545\,. \quad (5)$$

Thus, the fat content of the whole milk from which the 2% milk came from (bearing the label I derived my information from) was a little more than 3% milk fat by volume.