Math Diversion Problem 912

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November 19, 2025

Physical concepts are free creations of the human mind, and
are not, however it may seem, uniquely determined
by the external world.
— Albert Einstein

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Source: https://www.youtube.com/watch?v=KCobtZ05fE0
Title: This Logarithm Puzzle Broke the Internet.
 Presenter: Mental Math
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1 The Problem

Given the relation

$$m - n\log_3 2 = 10\log_9 6\,, \tag{1}$$

solve for integers m, n.

2 The Preparation

Here are some basic results about logarithms that might help.

$$\log_a bc = \log_a b + \log_a c\,, \tag{2a}$$

$$\log_a a^w = w\,, \tag{2b}$$

$$\log_{a^2} b = \tfrac{1}{2}\log_a b\,, \tag{2c}$$

$$\log_a b = \frac{\log_c a}{\log_c b}\,, \tag{2d}$$

$$\log_a b^w = w\log_a b\,, \tag{2e}$$

$$a^{\log_a w} = w\,. \tag{2f}$$

3 The Solution

We begin by rewriting (1) as

$$m - n\log_3 2 = 10(\tfrac{1}{2}\log_3 6) = 5\log_3 6\,, \tag{3}$$

where we used (2c). Then, using (2a), we get that

$$\log_3 6 = \log_3(2 \cdot 3) = \log_3 2 + \log_3 3 = \log_3 2 + 1\,, \tag{4}$$

where we also used (2b).

On plugging this result into (3) and then simplifying, we get

$$m - n\log_3 2 = 5\log_3 2 + 5\,. \tag{5}$$

With a little more algebra, we have that

$$m - 5 = 5\log_3 2 + n\log_3 2\,, \tag{6}$$

or better yet

$$m - 5 = \log_3 2^5 + \log_3 2^n\,, \tag{7}$$

where we used (2e).

Next, we use (2a) to get,

$$m - 5 = \log_3(2^5 2^n) = \log_3(2^{5+n})\,. \tag{8}$$

On raising 3 to the last equation, we get

$$3^{m-5} = 3^{\log_3(2^{5-n})} = 2^{5-n}\,, \tag{9}$$

where we used (2f).

Now, since m, n can only be integers, we must set these two exponents equal to zero, that is

$$m - 5 = 0 \quad \text{and} \quad 5 + n = 0\,, \tag{10}$$

from which we get

$$m = 5 \quad \text{and} \quad n = -5\,. \tag{11}$$