Math Diversion Problem 914

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First things first...But not necessarily in that order.
— Doctor Who

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Source: https://www.youtube.com/watch?v=edcp4YnE1zA
Title: if a^2+b^2+1/b^2+1/a^2=4 then ...
 Presenter: Adee Institute
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1 The Problem

Given the relation

$$a^2 + b^2 + \frac{1}{a^2} + \frac{1}{b^2} = 4\,, \tag{1}$$

what is the value of

$$\phi = a^2 + b^2\,? \tag{2}$$

2 The Solution

We begin by rewriting (1) as

$$a^2 + \frac{1}{a^2} = \alpha\,, \tag{3a}$$

$$b^2 + \frac{1}{b^2} = \beta\,, \tag{3b}$$

$$\alpha + \beta = 4\,. \tag{3c}$$

Now, it's quite obvious that if we swap a and b in (1) that the relation stays the same, thus a and b enter the problem symmetrically. That implies that if we swap a and b in (3a) and (3b) that we must get

$$b^2 + \frac{1}{b^2} = \alpha\,, \tag{4a}$$

$$a^2 + \frac{1}{a^2} = \beta\,, \tag{4b}$$

$$\alpha + \beta = 4\,. \tag{4c}$$

On comparing these two sets of equations, we get in the first place that $\alpha = \beta$. And then on including (4c), we must conclude that

$$\alpha = \beta = 2\,. \tag{5}$$

Now we return to

$$a^2 + \frac{1}{a^2} = 2\,. \tag{6}$$

If we multiply this equation through by a^2 and do a bit of algebra, we have that

$$(a^2 - 1)^2 = 0\,, \tag{7}$$

which has solution

$$a^2 = 1\,. \tag{8}$$

But because of the symmetry we already discussed, we also know that

$$b^2 = 1\,. \tag{9}$$

And finally we find that

$$\phi = a^2 + b^2 = 2\,. \tag{10}$$