Math Diversion Problem 918

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> The Axiom of Choice is obviously true; the Well Ordering
> Principle is obviously false; and who can tell
> about Zorn's Lemma?
> — Jerry Bona

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Source:https://www.youtube.com/watch?v=Hx761RqE9Iw
Title: Netherlands | Can you solve this??
 Presenter: Mr-Mathologer
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1 The Problem

Given the relation

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{x+y}\,, \tag{1}$$

solve for

$$\eta=\frac{x}{y}\,. \tag{2}$$

2 Solution

Eq. (1) can be rewritten in the form

$$\frac{y}{x}+1=\frac{y}{x+y}\,, \tag{3}$$

or in terms of η:

$$\eta^{-1}+1=\frac{1}{\eta+1}\,. \tag{4}$$

With a little algebra, we have that

$$\eta^2+\eta+1=0\,. \tag{5}$$

The solution to η by the quadratic formula is

$$\eta=\frac{-1\pm\sqrt{1-4}}{2}\,, \tag{6}$$

or

$$\eta=\frac{-1\pm i\sqrt{3}}{2}\,. \tag{7}$$