Math Diversion Problem 920

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The secret to perseverance is to just keep doing it.
— The Author

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Source: The Ether of Great Mathematical Ideas
Title: A divisor of 3 positive consecutive integers
 Presenter: Patrick
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1 The Problem

Show that 3! is a divisor of any 3 positive consecutive integers.

2 Solution

Since $3! = 6$ we can refashion the problem as to show that 6 is a divisor of any 3 positive consecutive integers. The proof is simple so long as we remember two basic facts of the positive integers. 1) Starting at zero, every other increasing integer is an even number, and thus is divisible by 2. All the other positive integers are odd numbers. Thus, every triple of consecutive integers will contain at least one even number, so that the triple product will always be divisible by 2.

2) For ease of visualization, think of highlighting every positive integer that is a multiple of three. Each of these is of the form $3k$ for some $k \in \mathbb{Z}^+$.

Now we turn to cases.

Case a) The three consecutive integers have the form

$$(3k-2)(3k-1)(3k)\,. \tag{1}$$

This clearly has a factor divisible by 3.

Case b) The three consecutive integers have the form

$$(3k-1)(3k)(3k+1)\,. \tag{2}$$

This also clearly has a factor divisible by 3.

Case c) [The final case.] The three consecutive integers have the form

$$(3k)(3k+1)(3k+2)\,. \tag{3}$$

This also clearly has a factor divisible by 3.

So, we have exhausted the cases and found that each case has both a factor of 2 and a factor of 3. Therefore, we have shown that for each triple product of three consecutive positive integers, the result is divisible by 3!.