

Math Diversion Problem 923

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Solving math puzzles can enhance cognitive functions
such as problem-solving, logical reasoning, and
memory, contributing to overall brain health.
— Daniel Levitin (renowned neuroscientist)

Source: Chemical Principles: Quest for Insight, 3rd Ed

Title: Problem 3. p. F49 F.7

Presenter: Atkins and Jones

1 Problem

In an experiment, 4.14 grams of phosphorus combined with chlorine to produce 27.8 grams of a white solid compound. What is the empirical formula of the white compound?

2 Solution

Notation: The empirical formula of the white compound has the general form of P_aCl_b , where a, b are relatively prime positive integers, representing the moles of phosphorus and chlorine, respectively. Our job is to find a and b .

Note: Let the grams per mole of P and Cl be, respectively, $[P]$ and $[Cl]$.

The key to solving this problem is to realize that the “algebraic ratio” of the grams of P to the grams of the compound must equal the ratio of the actual wt of P to the actual wt of the compound, that is,

$$\frac{a[P]}{a[P] + b[Cl]} = \frac{4.14 \text{ g}}{27.8 \text{ g}}. \quad (1)$$

We can do similarly for chlorine:

$$\frac{b[Cl]}{a[P] + b[Cl]} = \frac{27.8 \text{ g} - 4.14 \text{ g}}{27.8 \text{ g}} = \frac{23.66 \text{ g}}{27.8 \text{ g}}. \quad (2)$$

To simplify, divide (2) by (1) to get¹

$$\frac{b[\text{Cl}]}{a[\text{P}]} = \frac{23.66}{4.14} . \quad (3)$$

Therefore,

$$\frac{b}{a} = \frac{23.66}{4.14} \frac{[\text{P}]}{[\text{Cl}]} = 5.715 \frac{30.97 \text{ g}\cdot\text{mol}^{-1}}{35.46 \text{ g}\cdot\text{mol}^{-1}} \approx \frac{5}{1} . \quad (4)$$

So, the empirical formula of the white compound is PCl_5 .

¹I could have alternatively divided (1) by (2) but that would have produced a number less than 1. But, either way would work.