Math Diversion 924

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Source: The Ether of Great Mathematical Ideas
 Presenter: Patrick
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1 The Problem

Given the relation

$$\sqrt{x+1}+\sqrt{x-1}=2\,, \tag{1}$$

find the value of x.

2 The Solution

So, I chose as my 'first unipode' and follow-up:

$$a=\sqrt{x+1}u_+ +\sqrt{x-1}u_- \tag{2a}$$

$$=\frac{1}{2}\left[\sqrt{x+1}+\sqrt{x-1}\right]+\frac{1}{2}\left[\sqrt{x+1}-\sqrt{x-1}\right]u \tag{2b}$$

$$=1+\frac{1}{2}\beta u\,, \tag{2c}$$

where $\beta\equiv\sqrt{x+1}-\sqrt{x-1}$. By squaring this, we get

$$a^2=1+\frac{1}{4}\beta^2+\beta u\,. \tag{3}$$

On squaring a in (2a), we have that

$$a^2=(x+1)u_+ +(x-1)u_- =x+u\,. \tag{4}$$

On comparing a^2 between (3) and (4), we have that

$$x=1+\frac{1}{4}\beta^2,\quad \beta=1\,, \tag{5}$$

which gives us

$$x=\frac{5}{4}\,, \tag{6}$$