Math Diversion Problem 931

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Thanksgiving isn't Thanksgiving until
you've said thanks.
— Pastor Price

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Source: Chemical Principles: Quest for Insight, 3rd Ed
Title: Problem 7. p. F61 Ex. H.17
  Presenter: Atkins and Jones
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1 Problem

Phosphorus and oxygen react to form two different phosphorus oxides. The mass percentage of phosphorus in one of these is 43.64%; in the other, it is 56.34%.

(a) Write the empirical formula for each phosphorus oxide.

(b) The molar mass of the former oxide is 283.33 g mol^{-1} and that of the latter is 219.88 g mol^{-1}. Determine the molecular formulas and name each oxide.

2 Solution

First, some notation. Similar to what we did in Problem 3, we let [P] and [O] represent the molar masses of phosphorus and oxygen, respectively.

(a) So, we need to find relatively prime integers x, y for $\mathrm{P}_x\mathrm{O}_y$. To get the ratio of x and y, we can use that the ratio of the "algebraic mass" of phosphorus to the whole compound is equal to the actual ratio of their corresponding masses. Then,

$$\frac{x[\mathrm{P}]}{x[\mathrm{P}] + y[\mathrm{O}]} = .4364\,. \tag{1}$$

To help simplify, set $\alpha = y/x$ which gives us

$$\frac{[\mathrm{P}]}{[\mathrm{P}] + \alpha[\mathrm{O}]} = .4364\,. \tag{2}$$

So, with [P] = 30.97 and [O] = 16.00, then $\alpha = 2.5 = 5/2$. Therefore, the first empirical formula of 'phosphorus oxide' is P_2O_5.

By similar reasoning for the second compound, we get for its empirical formula P_2O_3.

Now, back to P_2O_5. To derive the actual chemical formula of this 'phosphorus oxide' compound, we need the appropriate multiplication factor γ, which we get from the following ratio:[1]

$$\begin{aligned}\gamma &= \frac{\text{Actual}\ \ \text{g}\cdot\text{mol}^{-1}}{\text{Empirical}\ \ \text{g}\cdot\text{mol}^{-1}} \\ &= \frac{283.88\,\text{g}\cdot\text{mol}^{-1}}{2(30.97)\,\text{g}\cdot\text{mol}^{-1} + 5(16)\,\text{g}\cdot\text{mol}^{-1}} \\ &= \frac{283.88}{141.94} \\ &\approx 2\,. \end{aligned} \tag{3}$$

Hence, the true molecular formula of the compound must be P_4O_{10}. The second case follows similarly.

[1]This γ is not standard. I had to choose something, so I chose it.