Math Diversion Problem 941

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December 1, 2025

First things first...But not necessarily in that order.
— Doctor Who

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Source: The Ether of Great Mathematical Ideas
Title: A Coin Problem
  Presenter: Patrick
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1 Problem

A jar contains a collection of less than 40 coins of two types, Type1 and Type2, valuing \$4.85. The possible coin types are penny, nickel, dime, and quarter. Find the types of coins and the number of each type of coin, if the following claims are true:

a) The percentage of Type1 coins by number is 41.37931%, and

b) The percentage of Type1 coins by value is 12.371134%.[1]

2 Solution

We begin with a diagram:

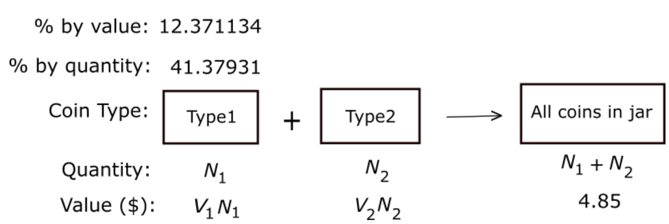

Figure B1. We add to the information in the diagram that the total number of coins is less than 40, or $N_1 + N_2 < 40$. V_1 and V_2 are the values per unit coin.

[1] These percentages are approximate values.

Now, using the information in the diagram, we present the obvious 'total equals the sum of its parts':

$$V_1N_1 + V_2N_2 = 4.85\,. \tag{1}$$

Next, we can convert the information in claim a) to

$$\frac{N_1}{N_1 + N_2} = 0.4137931\,, \tag{2}$$

and, from claim b)

$$\frac{V_1N_1}{V_1N_1 + V_2N_2} = 0.12371134\,. \tag{3}$$

To solve this system, let's begin with a simplification of (2) by dividing both numerator and denominator on the left hand side by N_1:

$$\frac{N_1}{N_1 + N_2} \to \frac{1}{1+x} = 0.4137931\,, \tag{4}$$

where $x = N_2/N_1$. Then we can use Wolframalpha (or some other means) to solve (4) for x:

$$x = \frac{N_2}{N_1} = 1.41667\,. \tag{5}$$

Next, we can do something similar for (3) which we did for (2), by dividing through by V_1N_1.

$$\frac{V_1N_1}{V_1N_1 + V_2N_2} = \frac{1}{1+xy} = \frac{1}{1+1.41667y} = 0.12371134\,, \tag{6}$$

where $y = V_2/V_1$. Solving for y, we get

$$y \approx 5\,. \tag{7}$$

Therefore, we take V_2 to be exactly five times V_1. Thus, we have two choices: Either the Type1 coin is a penny and the Type2 coin is a nickel, or the Type1 coin is a nickel and the Type2 coin is quarter. But the former case is ruled out because there is no combination of pennies and nickels that will equal \$4.85 if the total number of those coins is less than 40. Therefore,

$$V_1 = \$0.05 \quad \text{and} \quad V_2 = \$0.25\,. \tag{8}$$

Hence, (1) becomes

$$0.05N_1 + 0.25N_2 = 4.85\,. \tag{9}$$

On solving this last equation simultaneously with (2), we get

$$N_1 \approx 12\,, \quad N_2 \approx 17\,. \tag{10}$$

Therefore, we conclude that there are 12 nickels and 17 quarters in the jar.