Math Diversion 949

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> Conformal infinity is a very fascinating
> topic in relativity.
> — Tobias Osborne

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Source https://www.youtube.com/watch?v=_WMaStdei2g
Title: An logarithm problem
 Presenter: NonsoMaths
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1 The Problem

Given the relation

$$\log\left(\frac{x^{1/x}}{x^{1/(x+1)}}\right) = \frac{1}{5050}\,, \tag{1}$$

and where the logarithm is base 10, find the integer value of x.

2 Solution

Leaving the algebraic details to the reader, we have that

$$\frac{x^{1/x}}{x^{1/(x+1)}} = x^{1/x(x+1)}\,. \tag{2}$$

Thus (1) becomes

$$\log\left(x^{1/x(x+1)}\right) = \frac{1}{5050}\,. \tag{3}$$

At this point, it seemed logical enough to remove the logarithm; so I raised 10 to the power of this last equation, to get

$$x^{1/x(x+1)} = 10^{\frac{1}{5050}}\,. \tag{4}$$

Now it seems reasonable to make a variable substitution:

$$x = 10^y\,. \tag{5}$$

So now (4) becomes

$$10^{y/10^y(10^y+1)} = 10^{\frac{1}{5050}} \,. \tag{6}$$

On equating exponents and inverting, we have that

$$\frac{10^y(10^y+1)}{y} = 5050 \,. \tag{7}$$

I think that the best way to make progress at this point is to expand 5050:

$$5050 = \tfrac{1}{2}(100)(100+1) \,, \tag{8}$$

where I leave the details of the expansion to the reader. So, on substituting this last result into (7), we get

$$\frac{10^y(10^y+1)}{y} = \tfrac{1}{2}(100)(100+1) \,. \tag{9}$$

Clearly, the solution for y is $y = 2$. That means that the solution for x is:

$$x = 10^y = 10^2 = 100 \,. \tag{10}$$