Math Diversion 951

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Easy to criticize, more difficult to be correct.
— Charlie Chan

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Title: Question 521012
 Presenter: Patrick
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1 The Problem

Two Kleaning ladies company needs a 50% bleach mixture solution. they make 12 liter at a time from a 40% bleach solution and a 70% bleach solution. How many liters do they need of each?

2 Solution

A common sense way to make a 50% bleach mixture is by adding one part of bleach to one part of water. A 40% bleach mixture can be made directly by adding 4 parts of bleach to 6 parts of water. And, 70% bleach mixture can be made directly by adding 7 parts of bleach to 3 parts of water.

But our clever cleaners want to redeem some of the bleach mixes they already have on hand.

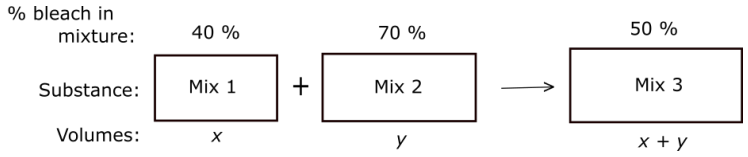

Figure 7. We've already shown the conservation of volumes.

Balancing for bleach, we have that

$$(.40)(x) + (.70)(y) = (.50)(x + y)\,, \tag{1a}$$

which can be simplified by the substitution $\lambda = x/y$ and multiplying through by 10, yielding us

$$4\lambda + 7 = 5\lambda + 5\,, \tag{1b}$$

which has solution $\lambda = 2$. Therefore, for every 2 parts of the 40% solution we add 1 part of the 70% solution to obtain 3 parts 50% solution. Now, let's prove that the ratio of bleach to water in Mix 3 is 1:1.

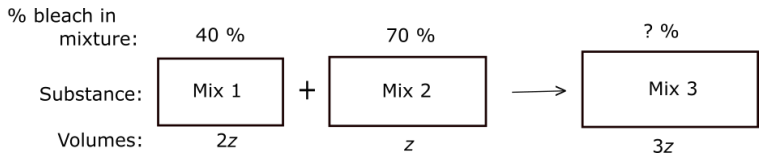

Figure 8. We have redone the last figure with the required ratio $x : y$ included for us. z is an arbitrary positive number of any volume unit.

Balancing for bleach, we have that

$$(.40)(2z) + (.70)(z) = 1.5z\,, \tag{2a}$$

and balancing for water, we have that

$$(.60)(2z) + (.30)(z) = 1.5z\,. \tag{2b}$$

Therefore, on taking the ratio of bleach to water, we get $1 : 1$, which is what we claimed was the correct way to make 50% bleach from scratch.