Math Diversion 952

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No mystery is closed to an open mind.
— Tim White
Sightings TV show

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Source: https://www.youtube.com/watch?v=UDbTLp77rfU
Title: A tricky math olympiad exam
 Presenter: Higher Mathematics
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1 Problem

Given the odd-looking relation

$$1^x = 8\,, \tag{1}$$

solve for x.

2 Solution

Obviously, over the real numbers, (1) has no solution, so we must look for solutions over the complex numbers. Hence, we must change the face of the problem to this

$$(e^{2\pi ik})^x = 8 \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{2}$$

This works because

$$e^{2\pi ik} = 1 \quad \text{for all} \quad k \in \mathbb{Z}\,. \tag{3}$$

So, (2) becomes

$$e^{2\pi ikx} = 2^3 \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{4}$$

Next, we take the natural logarithm across this equation, to get

$$2\pi ikx = 3\ln 2 \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{5}$$

On solving for x, we have that

$$x = \frac{3\ln 2}{2\pi ik} = -i\frac{3\ln 2}{2\pi k} \quad \text{where} \quad k \in \mathbb{Z}\,. \tag{6}$$