Math Diversion 954

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> Mathematics is the art of reducing any
> problem to linear algebra.
> — William Stein

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Source: The Ether of Great Mathematical Ideas
Title: A Lambert Identity
 Presenter: Patrick
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1 Problem

Prove the Lambert identity:

$$W(x^{x+1}\ln x) = x\ln x\,. \tag{1}$$

2 Solution

We begin with another identity:

$$x^x = e^{x\ln x}\,. \tag{2}$$

Next, multiply through by $x\ln x$:

$$(x\ln x)x^x = (x\ln x)e^{x\ln x}\,. \tag{3}$$

Or,

$$x^{x+1}\ln x = (x\ln x)e^{x\ln x}\,. \tag{4}$$

Now take the Lambert W function across this, and we have that

$$W(x^{x+1}\ln x) = x\ln x\,. \tag{5}$$