Math Diversion 956

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> Young men should prove theorems, old
> men should write books.
> — G. H. Hardy

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Source: https://www.youtube.com/watch?v=_y-64fzCl1o
Title: A Problem from A Primer For Mathematics Competitions
 Presenter: SyberMath
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1 Problem

Given the relation

$$7a + 13b = 1000\,, \tag{1}$$

find a few pairs of integer values (a, b) that satisfy this relation, where we assume $a, b > 0$.

2 Solution

This solution is all about modular arithmetic. So, let's take the given equation mod 7 or 13. I choose 7:

$$13b \equiv 1000 \pmod 7\,, \tag{2}$$

where the first term $7a$ dropped out because it's congruent to zero mod 7.

Next, we reduce both 13 and 100 mod 7, to get

$$6b \equiv 6 \pmod 7\,, \tag{3}$$

The possible values of b that can solve (3) are

$$b = 1, 8, 15, 22, 29, 36, 43, 50, 57, 64, 71, 78, \dots\,. \tag{4}$$

So, $b = 1$ is the obvious minimum value, and then we can add to that value any multiple of 7. However, we can't go arbitrarily high in values or we'll break the original constraint (1). Actually, 78 is already too high because $13 \times 78 = 1014$, which doesn't leave any room for a non-negative value for a.

Anyway, on trying $b = 71$, we get $a = 11$, so that as a solution we have that $(11, 71)$. We also find solution $(24, 64)$ and $(76, 36)$.