

Math Diversion 958

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When I give this talk to a physics audience, I
remove the quotes from my 'Theorem'.
— Brian Greene

Source: <https://www.youtube.com/watch?v=caSZtP7YnFE>
Title: Harvard University Interview Test
Presenter: Spencer's Academy

1 Problem

Given the relations

$$ab = 10, \tag{1a}$$

$$bc = 20, \tag{1b}$$

$$ac = 30, \tag{1c}$$

$$\phi = a + b + c, \tag{1d}$$

solve for ϕ , where we assume $a, b, c > 0$.¹

2 Solution

From (1a)–(1c), we have that⁶

$$abc = 10c, \tag{2a}$$

$$abc = 20a, \tag{2b}$$

$$abc = 30b. \tag{2c}$$

¹You can also solve for negative values if you want, but it doesn't add much of interest to the solution.

After a little division, we can add the resulting equations together, to get

$$\begin{aligned}\phi &= a + b + c \\ &= \frac{abc}{20} + \frac{abc}{30} + \frac{abc}{10} \\ &= \frac{abc}{10} \left(1 + \frac{1}{2} + \frac{1}{3}\right) \\ &= \frac{11}{60} abc.\end{aligned}\tag{3}$$

Next, on multiplying Equations (1a)–(1c), we have that

$$(abc)^2 = 6000.\tag{4}$$

Hence,

$$abc = \sqrt{6000}.\tag{5}$$

On substituting this value into (3), we have that

$$\phi = \frac{11}{60} \sqrt{6000} = 11\sqrt{\frac{5}{3}},\tag{6}$$

which is what WolframAlpha got.