Math Diversion 960

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You cannot ask us to take sides against arithmetic.
— Winston Churchill

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Source: https://www.youtube.com/watch?v=amu87QlQrP0
Title: Harvard University Entrance math Examination question
 Presenter: JJ ONLINE MATHS CLASS
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1 Problem

Given the relation

$$27^x - 3^x = \sqrt{12}\,, \tag{1}$$

find the real solutions for x. (If you are interested in all the possible solutions for x, see WolframAlpha.)

2 Solution

We'll begin by rewriting (1) as

$$(3^x)^3 - 3^x = 2\sqrt{3}\,, \tag{2}$$

and then introducing the variable

$$3^x = \sqrt{3}\,y\,. \tag{3}$$

Then using (3), (2) becomes, after a bit of algebraic simplification:

$$3y^3 - y - 2 = 0\,. \tag{4}$$

This equation is simple enough to try small values for y to see what might work. It turns out that $y = 1$ works. So, the x value corresponding to this y value is

$$x = \frac{1}{2}\,. \tag{5}$$

Technically, we're finished, but if you're interested in the values of the other two solutions (which are complex), then (4) factors to

$$(y-1)(3y^2+3y+2)=0\,. \tag{6}$$

So, solve for the complex roots of

$$3y^2+3y+2=0\,, \tag{7}$$

and then use those roots in (3). (Hint: You'll need to use logarithms.)