Math Diversion 963

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I love it when a plan comes together.
— Hannibal Smith, *The A-Team*

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Source: https://www.algebra.com/algebra
Title: Question 364411: A Mixed-Rate Problem
 Presenter: Patrick
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1 Problem

Two alloys contain silver and copper in the ratios 3 : 1 and 5 : 3, respectively. The alloys are mixed to get a third alloy. The possible ratio of silver to copper in the third alloy is?

2 Solution

First, I believe that the question given us in the problem is ill-posed. It should state: Determine the ratio of silver to copper in the final alloy as a function of the arbitrary amounts of the two alloys mixed together.

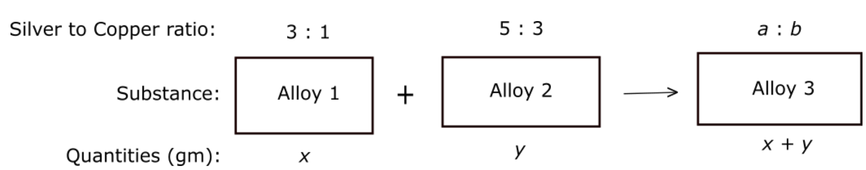

Figure 1. The value we need to solve for is a/b.

Second, I've made the assumption that the ratios were given by weight (or mass). I've chosen grams, but any unit of weight (mass) will do because in the ratio we're looking for, the units will cancel out.

Notice that the figure reflects the fact that the problem as stated makes no restriction on the relative amounts of silver and copper that can be mixed together. Therefore, it's reasonable to assume right now that x and y variables are independent and that they will show up in the answer on an equal footing.

Now, to get the ratio of a to b, we begin by balancing on both the silver and copper components of the process.

$$\text{Balance on silver:} \quad \frac{3}{4}\,x + \frac{5}{8}\,y = \frac{a}{a+b}\,(x+y)\,, \tag{1a}$$

$$\text{Balance on copper:} \quad \frac{1}{4}\,x + \frac{3}{8}\,y = \frac{b}{a+b}\,(x+y)\,. \tag{1b}$$

Now, we let $\lambda = a/b$, as we have done in the past. Then we take (1a) $\div$ (1b), and simplify to get

$$\frac{\frac{3}{4}\,x + \frac{5}{8}\,y}{\frac{1}{4}\,x + \frac{3}{8}\,y} = \lambda\,. \tag{2}$$

One last simplification gives us

$$\lambda = \frac{6x+5y}{2x+3y}\,. \tag{3}$$

Let's go one step further and ask ourselves how much copper should be added to 100 grams of silver to produce an alloy of quality sterling silver? First, we need to know that sterling silver has quality defined to be 92.5% silver against all other metals, in this case copper.

Plugging-in the values into (3), we get $\lambda = 92.5/7.5$, so

$$92.5/7.5 = \frac{600+5y}{200+3y}\,. \tag{4}$$

which has the unrealistic solution $y = -58.3333$. This means that we'll never get sterling silver by this combination of alloys, and it suggests that we have tried an out-of-bounds value for λ. Let's investigate this.

If we take $y = 0$, what λ do we get? We get, $\lambda = 3/1$, yielding a percentage of 75%. If we take $x = 0$, what λ do we get? We get, $\lambda = 5/3$, yielding a percentage of 62.5%. Therefore, it's reasonable to believe that the λ values will vary from a low of 5/3 to a high of 3/1. Let's now prove this.

With just a little effort, Equation (3) can be rewritten as

$$(3\lambda - 5)y = (6 - 2\lambda)x\,. \tag{5}$$

Neither x nor y can be negative, and, since we've already tested what happens when either one is zero, that leaves us with the constraint on their coefficients that they be both positive or both negative.

Case 1) Both positive: $(3\lambda - 5) > 0$ and $(6 - 2\lambda) > 0$. This gives solutions for λ given by

$$\frac{5}{3} < \lambda < \frac{3}{1}\,. \tag{6}$$

Case 2) Both negative: $(3\lambda - 5) < 0$ and $(6 - 2\lambda) < 0$. This gives no solutions for λ.