Math Diversion 964

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> You either get control of your lusts and feelings of entitlement, or they will get control of you.
> — The Author

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Source: https://www.youtube.com/watch?v=uRtalj-oqKk
Title: Think You Know Logs?
 Presenter: NonsoMaths
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1 Problem

Given the relation

$$15^{\log_5 3} = \frac{1}{x^{\log_5(9x)+1}}\,, \tag{1}$$

solve for x.

2 Solution

Let's begin by simplifying the LHS.

$$\begin{aligned} 15^{\log_5 3} &= 3^{\log_5 3} 5^{\log_5 3} && \text{(2a)} \\ &= 3^{\log_5 3} 3 && \text{(2b)} \\ &= 3^{\beta+1}\,, && \text{(2c)} \end{aligned}$$

where

$$\beta \equiv \log_5 3\,. \tag{3}$$

Next, we look at the RHS of (1), while using the variable substitution,

$$x = 3^{\alpha}\, : \tag{4}$$

$$
\begin{aligned}
x^{\log_5(9x)+1} &= 3^{\alpha[\log_5(9)+\log_5(3^\alpha)+1]} && (5a)\\
&= 3^{\alpha[\log_5(3^2)+\log_5(3^\alpha)+1]} && (5b)\\
&= 3^{\alpha[2\log_5(3)+\alpha\log_5(3)+1]} && (5c)\\
&= 3^{\alpha[2\beta+\alpha\beta+1]} && (5d)\\
&= 3^{[\alpha(2\beta+1)+\alpha^2\beta]}\,. && (5e)
\end{aligned}
$$

So, upon setting the LHS of (1) equal to the RHS (1), we have that

$$3^{\beta+1} = 3^{-[\alpha(2\beta+1)+\alpha^2\beta]}\,. \tag{6}$$

On setting the exponents equal and then applying some algebra, we get the following quadratic in variable α:

$$\beta\alpha^2 + (2\beta+1)\alpha + (\beta+1) = 0\,. \tag{7}$$

Using the quadratic formula for α, we get

$$\alpha = \frac{-(2\beta+1) \pm \sqrt{(2\beta+1)^2 - 4(\beta)(\beta+1))}}{2\beta}\,. \tag{8}$$

Fortunately, this simplifies tremendously.

$$\alpha = -1,\ -1-\beta^{-1}\,. \tag{9}$$

For $\alpha = -1$, we get for x:

$$x = 3^{-1} = \frac{1}{3}\,. \tag{10}$$

To deal with the other root, I leave it as a lemma for the reader to prove that

$$\log_a b = (\log_b a)^{-1}\,. \tag{11}$$

Anyway, from this we can conclude that

$$\beta^{-1} = \log_3 5\,. \tag{12}$$

For $\alpha = -1 - \beta^{-1} = -1 - \log_3 5$, we get for x:

$$
\begin{aligned}
x &= 3^{-1-\log_3 5} && (13a)\\
&= 3^{-1}3^{\log_3 5^{-1}} && (13b)\\
&= \frac{1}{3}\cdot\frac{1}{5} && (13c)\\
&= \frac{1}{15}\,. && (13d)
\end{aligned}
$$