

Math Diversion 966

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If there is to be a brave new world, our generation is
going to have the hardest time living in it.
— Chancellor Gorkon (Star Trek VI,
The Undiscovered Country)

Source: <https://www.youtube.com/watch?v=aX4IHM0VcXk>
Title: A fancier way to solve this radical equation
Presenter: blackpenredpen

1 The Problem

Given the relation

$$\sqrt[3]{x-40} + \sqrt[3]{-x+3} = -1, \quad (1)$$

find the real values of x .

2 The Solution

So, I chose as my ‘first unipode’ and follow-up:

$$a = \sqrt[3]{x-40}u_+ + \sqrt[3]{-x+3}u_- \quad (2a)$$

$$= \frac{1}{2}[\sqrt[3]{x-40} + \sqrt[3]{-x+3}] + \frac{1}{2}[\sqrt[3]{x-40} - \sqrt[3]{-x+3}]u \quad (2b)$$

$$= -\frac{1}{2} + \frac{1}{2}Bu, \quad (2c)$$

where

$$B \equiv \sqrt[3]{x-40} - \sqrt[3]{-x+3}. \quad (3)$$

On cubing a in (2c), and then taking its scalar part, we have that

$$\langle a^3 \rangle_0 = \left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)B^2/4. \quad (4)$$

On cubing a in (2a), and then converting to its scalar and unipotent parts, we have that

$$a^3 = (x - 40)u_+ + (-x + 3)u_- \quad (5a)$$

$$= \frac{1}{2}(-37) + (\text{unipotent part})u. \quad (5b)$$

Now, the scalar part of this last equation must equal the expression in (4), hence

$$\frac{1}{2}(-37) = \left(-\frac{1}{2}\right)^3 + 3\left(-\frac{1}{2}\right)B^2/4. \quad (6)$$

This equation gives us two values for B :

$$B = +7, \quad -7. \quad (7)$$

I think that the simplest way to proceed is to add equations (1) and (3) together, to get (after a lot of algebra), for $B = +7$

$$x = 67. \quad (8)$$

And for $B = -7$, we get

$$x = -24. \quad (9)$$