

# Math Diversion 968

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Scarecrows and magic and other fatal fears do not bring  
people together. There is no substitute for soft caring  
and hard work; for self-respect and mutual love.  
— *The Outer Limits*, “Architects of Fear.”

Source: <https://www.youtube.com/watch?v=VwNwG3ddo6k>

Title: Germany | Can you solve this?

Presenter: SALogic

## 1 Problem

Given the relation

$$4^x = x^8, \tag{1}$$

solve for the real values of  $x$ .

## 2 Solution

Since we are only interested in the real solutions, we can extract the eighth root of (1), to get

$$(4^{1/8})^x = x. \tag{2}$$

Let's simplify this by making the substitution:

$$\beta = 4^{1/8}, \tag{3}$$

and for future reference, we add

$$\ln \beta = \frac{1}{4} \ln 2. \tag{4}$$

So, (2) becomes

$$\beta^x = x. \tag{5}$$

Or,

$$1 = x\beta^{-x}. \tag{6}$$

Or,

$$-x\beta^{-x} = -1. \quad (7)$$

Next, we take the Lambert W function base  $\beta$  to get<sup>1</sup>

$$-x = W_{(\beta)}(-1) = \frac{W_n(-1 \cdot \ln \beta)}{\ln \beta} \quad \text{where } n \in \mathbb{Z}. \quad (8)$$

Therefore,

$$x = -4 \frac{W_n(-\frac{1}{4} \ln 2)}{\ln 2} \quad \text{where } n \in \mathbb{Z}. \quad (9)$$

Again, we are interested in the real values for  $x$ , which means that  $n$  will be restricted to the values  $n = -1, 0$ .

WolframAlpha gives for  $n = 0$ :

$$-4(\text{ProductLog}[0, (-\ln(2))/4])/\ln(2) = 1.2396\dots$$

WolframAlpha gives for  $n = -1$ :

$$-4(\text{ProductLog}[-1, (-\ln(2))/4])/\ln(2) = 16.$$

Actually, WolframAlpha gave one more real solution, namely,  $x = -0.861345$ , but I don't know what  $n$  value that goes with.

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<sup>1</sup>This will take us to a lemma, which is explained in my PDF writeup on the Lambert W function.