

Math Diversion 972

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The ugliest curve you're ever likely to see in your life
is the *learning curve*. Deal with it.
— The Author

Source: <https://www.youtube.com/watch?v=euBnlvAanxY>
Title: France | Can you solve?
Presenter: Math Master TV

1 Problem

The following relation

$$\phi = (\sqrt{11} + \sqrt{5})^8 + (\sqrt{11} - \sqrt{5})^8, \quad (1)$$

has a simple (though large) integer value. Find it.

2 Solution

This can be solved in the unipodal algebra, so I'll do that just to see how the solution goes.

For my initial unipodal element, I'll start with

$$a = (\sqrt{11} + \sqrt{5}) u_+ + (\sqrt{11} - \sqrt{5}) u_-, \quad (2)$$

which is in the idempotent basis. Converted to the standard basis, we have that

$$a = \sqrt{11} + \sqrt{5} u. \quad (3)$$

Now, in order to interject ϕ into one of these equations, we need to raise (1) to the eighth power, getting

$$a^8 = (\sqrt{11} + \sqrt{5})^8 u_+ + (\sqrt{11} - \sqrt{5})^8 u_-, \quad (4)$$

which, in the standard basis is

$$\begin{aligned} a^8 &= \frac{1}{2}[(\sqrt{11} + \sqrt{5})^8 + (\sqrt{11} - \sqrt{5})^8] + \frac{1}{2}[(\sqrt{11} + \sqrt{5})^8 - (\sqrt{11} - \sqrt{5})^8]u \\ &= \frac{1}{2}\phi + \frac{1}{2}Bu, \end{aligned} \quad (5)$$

where

$$B \equiv (\sqrt{11} + \sqrt{5})^8 - (\sqrt{11} - \sqrt{5})^8. \quad (6)$$

However, we won't need B .

Now, we have (3), which is a pure unipode (i.e., has only numbers in it) and we have a unipode in (5) that contains our unknown ϕ . We just need to find a 'simple' relation between them. Actually, we do! Let

$$\langle a^8 \rangle_0 = \langle a^8 \rangle_0, \quad (7)$$

where we'll substitute into the LHS the a^8 from (5) and into the RHS the a^8 from (3). Thus,

$$\langle \frac{1}{2}\phi + \frac{1}{2}Bu \rangle_0 = \langle (\sqrt{11} + \sqrt{5}u)^8 \rangle_0. \quad (8)$$

The LHS simplifies to just $\frac{1}{2}\phi$. When we expand the RHS, we're going to need the binomial coefficients provided by Pascal's Triangle, which are

$$\mathbf{1}, \quad 8 \quad \mathbf{28}, \quad 56, \quad \mathbf{70}, \quad 56, \quad \mathbf{28}, \quad 8, \quad \mathbf{1}. \quad (9)$$

But since the selectors have been specially setup to choose only the scalar components (those where u will be to an even power), we only need the bold entries. Thus,

$$\begin{aligned} \frac{1}{2}\phi &= (1)(\sqrt{11})^8 + (28)(\sqrt{11})^6(\sqrt{5})^2 + (70)(\sqrt{11})^4(\sqrt{5})^4 + (28)(\sqrt{11})^2(\sqrt{5})^6 + (\sqrt{5})^8 \\ &= 11^4 + (28)(11)^3(5) + (70)(11)^2(5)^2 + (28)(11)(5)^3 + (5)^4 \\ &= 14614 + 186340 + 211750 + 38500 + 625 \\ &= 451856. \end{aligned} \quad (10)$$

Therefore,

$$\phi = 903,712. \quad (11)$$