

Math Diversion 973

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Every big idea needs someone to defend it.
— Cybersecurity

Source: <https://www.youtube.com/watch?v=t6IzRCScKIc>

Title: Can $d^2y/dx^2=(dy/dx)^2$?

Presenter: blackpenredpen

1 Problem

Given the relation

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2, \quad (1)$$

solve for $y = y(x)$.

2 Solution

We begin with a substitution:

$$z = \frac{dy}{dx}, \quad (2)$$

then (1) becomes

$$\frac{dz}{dx} = z^2. \quad (3)$$

From this we can form an integral equation (separation of variables)

$$\int \frac{dz}{z^2} = \int dx, \quad (4)$$

from which we have that

$$-\frac{1}{z} = x + c_1, \quad (5)$$

On solving for z , we get

$$z = \frac{dy}{dx} = -\frac{1}{x + c_1}. \quad (6)$$

So the solution for y is

$$y(x) = -\ln(x + c_1) + c_2. \quad (7)$$