Math Diversion 974

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Dear Algebra, stop asking us to find your X,
she's not coming back.
— Woody Paige

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Source: https://www.youtube.com/watch?v=QFDxHYb1cDk
Title: Most Students Give Up on This Log Equation
 Presenter: NonsoMaths
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1 Problem

Given the relation

$$\log_{1/2}^2(4x) + \log_2\left(\frac{x^2}{8}\right) = 8\,, \tag{1}$$

find x.

2 Preface

Some lemmas to note:

$$\log_a B = \frac{\log_c B}{\log_c a}\,, \tag{2}$$

where c is an arbitrary positive real number. Also,

$$\log_a(AB) = \log_a(A) + \log_a(B)\,, \tag{3}$$

and

$$\log_a(A/B) = \log_a(A) - \log_a(B)\,. \tag{4}$$

Also,

$$\log_a a = 1\,. \tag{5}$$

Lastly,

$$\log_a B^y = y\log_a B\,, \tag{6}$$

Now, I won't be calling out these lemmas in what follows as they are used, so if you see a step you don't get, look back to these lemmas for guidance. Oh, just one more thing: $8 = 2^3$ and $2^7 = 128$.

3 Solution

The Given relation can be rewritten as

$$\left(\frac{\log_2(4x)}{\log_2 \frac{1}{2}}\right)^2 + \log_2 x^2 - \log_2 8 = 8\,, \tag{7}$$

which can then be cast as

$$\left(\frac{\log_2 4 + \log_2 x}{\log_2 2^{-1}}\right)^2 + 2\log_2 x - 3\log_2 2 = 8\,, \tag{8}$$

Now, let

$$y \equiv \log_2 x\,, \tag{9}$$

Then (8) becomes

$$(\log_2 2^2 + y)^2 + 2y - 3 = 8\,, \tag{10}$$

or rather

$$(y+2)^2 + 2y - 11 = 0\,, \tag{11}$$

We just need to place this quadratic in standard form:

$$y^2 + 6y - 7 = 0\,, \tag{12}$$

which, by inspection, can be factored as

$$(y-1)(y+7) = 0\,, \tag{13}$$

The value for x corresponding to $y = 1$ is $x = 2$, and the value for x corresponding to $y = -7$ is $x = 1/128$.