

Math Diversion 976

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Mental toughness is essential to success.

— Vince Lombardi

Source: The Ether of Great Mathematical Ideas

Title: An easy indefinite integral

Presenter: Patrick

1 Problem

Perform the indefinite integral

$$I = \int x^3 e^{x^2} dx. \quad (1)$$

2 Solution

Let's try an likely expression to differentiate first!

$$D_x(x^2 e^{x^2}) = 2x e^{x^2} + 2x^3 e^{x^2}. \quad (2)$$

Now, integrate through:

$$x^2 e^{x^2} = 2 \int x e^{x^2} dx + 2I. \quad (3)$$

On solving for I :

$$I = \frac{1}{2} x^2 e^{x^2} - \int x e^{x^2} dx. \quad (4)$$

But

$$\int x e^{x^2} dx = \frac{1}{2} \int e^{x^2} d(x^2) = \frac{1}{2} e^{x^2}. \quad (5)$$

On using this result in (4), we have that

$$I = \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C = \frac{1}{2} (x^2 - 1) e^{x^2} + C. \quad (6)$$